

Fragmentation functions of mesons in the Field–Feynman model

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Abstract. The fragmentation functions of the pion with distinction between $D_u^{\pi^+}$, $D_d^{\pi^+}$, and $D_s^{\pi^+}$ are studied in the Field–Feynman recursive model, by taking into account the flavor structure in the excitation of quark–antiquark pairs by the initial quarks. The obtained analytical results are compatible with the available empirical results. The framework is also extended to predict the fragmentation functions of the kaon with distinction between $D_s^{K^+}(z)$, $D_u^{K^+}(z)$, $D_s^{K^+}(z)$, and $D_d^{K^+}(z)$. This work gives a significant modification of the original model, and the predictions can be tested by future experiments on the fragmentation functions of the kaon.

1 Introduction

The productions of mesons are under investigation in various processes nowadays; therefore the fragmentation functions of mesons are among the useful basic quantities in high energy physics studies. Complete knowledge of the quark to meson fragmentation functions with detailed flavor structure is needed and will be more useful. In previous studies, the fragmentation functions of mesons are usually obtained from experimental data on e^+e^- annihilation process [1]. However, the situation has now been changed because the HERMES collaboration has published results of charge separated data for π^\pm production on the proton target [2]. By combining the HERMES data on semi-inclusive DIS (SIDIS) π^\pm -production with the singlet fragmentation function $D_\Sigma^{\pi^+}$, which is well determined from e^+e^- data, Kretzer, Leader, and Christova [3] obtained a complete set of fragmentation functions of the pion: $D_u^{\pi^+}$, $D_d^{\pi^+}$, and $D_s^{\pi^+}$, without making a specific assumption about the relation between favored and unfavored transitions, such as $D_d^{\pi^+}(z) = (1-z)D_u^{\pi^+}(z)$ in [4] and $D_d^{\pi^+}(z) = (1-z)^n D_u^{\pi^+}(z)$ (with $n = 2, 3, 4$) in [5]. The Kretzer–Leader–Christova (KLC) parameterization results of the experimental data are

$$D_u^{\pi^+}(z) = 0.689z^{-1.039}(1-z)^{1.241}, \quad (1)$$

$$D_d^{\pi^+}(z) = 0.217z^{-1.805}(1-z)^{2.037}, \quad (2)$$

$$D_s^{\pi^+}(z) = 0.164z^{-1.927}(1-z)^{2.886}. \quad (3)$$

Although the evolution behaviors can be calculated perturbatively, the actual form of the fragmentation

functions is non-perturbative. Therefore the fragmentation functions are helpful to elucidate the fundamental features of hadronization. The pioneering analysis by Field and Feynman [6, 7] provides a simple and lucid picture to understand the fragmentation for a quark into mesons based on a recursive principle, and has stimulated more sophisticated models like the string fragmentation model, implemented in Monte Carlo generation programs [1], as in the Lund model [8]. The Field–Feynman model is successful for parameterizing the fragmentation functions of the pion and the kaon with only two parameters. However, no distinction between $D_d^{\pi^+}(z)$ and $D_s^{\pi^+}(z)$ was made in the original model [6], for the sake of the reduction of the number of independent parameters. With the recent progress of making a clear distinction between $D_d^{\pi^+}(z)$ and $D_s^{\pi^+}(z)$, it becomes mature and necessary to consider the flavor structure of the unfavored fragmentation functions of mesons in the Field–Feynman model. The purpose of this paper is for the first time to make a distinction between $D_d^{\pi^+}(z)$ and $D_s^{\pi^+}(z)$ in the Field–Feynman model. It will be shown that this can be realized by adopting the flavor dependent parameters for the excitation of sea quark–antiquark pairs by the initial quarks. We will find that the obtained analytical results for $D_u^{\pi^+}$, $D_d^{\pi^+}$, and $D_s^{\pi^+}$ with only three parameters, are compatible with the recent KLC parameterization (1)–(3) based on experimental results. We will also extend the same framework to the fragmentation functions of the kaon and show that we can distinguish between $D_s^{K^+}(z)$, $D_u^{K^+}(z)$, $D_s^{K^+}(z)$, and $D_d^{K^+}(z)$ without introducing any additional parameters. Thus the predictions can be tested by future experiments on the fragmentation functions of the kaon.

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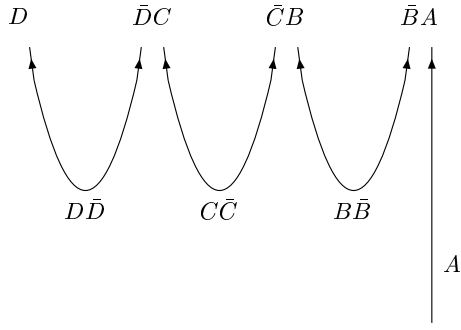


Fig. 1. Illustration of the Field–Feynman recursive model for the fragmentation of mesons. The initial quark A combines with \bar{B} to form the rank 1 “primary” meson with configuration $\bar{B}A$, and the remained quark B goes on the same way as the former quark A

The paper is organized as follows. In Sect. 2, we present a brief review of the Field–Feynman recursive model for the quark to meson fragmentation functions, and make a distinction between the parameters that can introduce the difference between $D_d^{\pi^+}(z)$ and $D_s^{\pi^+}(z)$. In Sect. 3, we adjust the parameters based on the KLC parameterization and the experimental results. We present the analytical results of $D_u^{\pi^+}$, $D_d^{\pi^+}$, and $D_s^{\pi^+}$, and compare them with the experimental results. In Sect. 4, we extend the framework to the fragmentation functions of the kaon, and make predictions of the four different fragmentation functions $D_s^{K^+}(z)$, $D_u^{K^+}(z)$, $D_s^{K^+}(z)$, and $D_d^{K^+}(z)$. Finally, we present a summary in Sect. 5.

2 A brief review of the recursive model

The Field–Feynman model [6] of the meson fragmentation is based on a recursive principle, as can be illustrated in Fig. 1. The model ansatz is based on the idea that an incoming quark “ A ” combines with an antiquark “ \bar{B} ” from a quark–antiquark pair “ $B\bar{B}$ ” which is produced from the color field excited by the incoming quark “ A ”. Then the meson “ $A\bar{B}$ ” is constructed, and is ranked as a 1st primary meson. The remaining quark “ B ” goes on the same way as the quark “ A ”, and combines with “ \bar{C} ” from a quark–antiquark pair “ $C\bar{C}$ ” which is excited by the quark “ B ”. It constructs the primary meson “ $B\bar{C}$ ”, ranked as a 2nd primary meson. Then the remaining quark “ C ” goes on the same way in the remaining cascade.

With $f(\eta)d\eta$ denoting the probability that the first rank 1 meson leaves a fractional momentum η to the remaining cascade, the function $f(\eta)$ is normalized by

$$\int_0^1 f(z)dz = 1. \quad (4)$$

Now let $F(z)dz$ be the probability of finding a meson (independent of rank) with fractional momentum z within dz in a quark jet. So $F(z)$ can be written by the recursive

principle

$$F(z) = f(1-z) + \int_0^1 \frac{d\eta}{\eta} f(\eta) F\left(\frac{z}{\eta}\right). \quad (5)$$

The first term comes from the ranked 1 primary meson. The second term calculates the probability of the production of a higher rank meson recursively. Field and Feynman [6] found a solution for the above integral equation with

$$zF(z) = f(1-z), \quad (6)$$

by choosing

$$f(z) \equiv (d+1)z^d. \quad (7)$$

With β_{ij} being the probability of finding the $q_j\bar{q}_j$ pair in the quark sea excited by the initial quark i , the normalization condition imposes that

$$\sum_{j=1}^{n_f} \beta_{ij} = 1. \quad (8)$$

Here, we add an i index which does not appear in the original model, and we will find that it is useful. The modification of this model is based on the difference between β_{uu} and β_{su} , which were assumed to be the same in the original Field–Feynman model for the sake of reducing the number of independent parameters. The original model predicts no distinction between $D_d^{\pi^+}$ and $D_s^{\pi^+}$, and the results in [3] show that this is not the realistic situation. Adopting $SU(2)$ symmetry between u and d quarks, we have

$$\beta_{iu} = \beta_{id} \equiv \beta_i; \quad (9)$$

we get

$$\beta_{is} = 1 - 2\beta_i. \quad (10)$$

For an initial quark of flavor q , the mean number of fragmented mesons with configuration $a\bar{b}$ at z is given, in analogy of (5), by

$$P_q^{a\bar{b}}(z) = \delta_{aq}\beta_{qb}f(1-z) + \int_z^1 \frac{d\eta}{\eta} f(\eta)\beta_{qc}P_c^{a\bar{b}}(z/\eta). \quad (11)$$

The mean number of meson states averaged over all quarks is

$$P_{\langle q \rangle}^{a\bar{b}}(z) = \sum_c \beta_{qc}P_c^{a\bar{b}}(z), \quad (12)$$

so that

$$P_{\langle q \rangle}^{a\bar{b}}(z) = \beta_{qa}\beta_{qb}f(1-z) + \int_z^1 \frac{d\eta}{\eta} f(\eta)\beta_{qc}P_{\langle c \rangle}^{a\bar{b}}(z/\eta). \quad (13)$$

Comparing with (5), this yields

$$P_{\langle q \rangle}^{a\bar{b}}(z) = \beta_{qa}\beta_{qb}F(z). \quad (14)$$

Substituting back into (13), one finds

$$P_q^{a\bar{b}}(z) = \delta_{aq}\beta_{qb}f(1-z) + \beta_{qa}\beta_{qb}\bar{F}(z), \quad (15)$$

where

$$\begin{aligned}\bar{F}(z) &\equiv F(z) - f(1-z) = \left(\frac{1}{z} - 1\right) f(1-z) \\ &= (d+1)z^{-1}(1-z)^{d+1}.\end{aligned}\quad (16)$$

The fragmentation function is

$$D_q^h(z) = \sum_{ab} \Gamma_{ab}^h P_q^{a\bar{b}}(z), \quad (17)$$

where Γ_{ab}^h is the probability of a meson h with configuration $a\bar{b}$. For example, $\Gamma_{ud}^{\pi^+} = 1$ and $\Gamma_{uu}^{\pi^0} = \Gamma_{dd}^{\pi^0} = 1/2$. Combining (11) and (13), we obtain

$$D_q^h(z) = A_q^h f(1-z) + B_q^h \bar{F}(z), \quad (18)$$

where

$$A_q^h = \sum_b \Gamma_{qb}^h \beta_{qb}, \quad (19)$$

$$B_q^h = \sum_{a,b} \beta_{qa} \Gamma_{ab}^h \beta_{qb}. \quad (20)$$

Then the fragmentation functions of $D_u^{\pi^+}$, $D_d^{\pi^+}$, and $D_s^{\pi^+}$ can be written as

$$D_u^{\pi^+}(z) = \beta_u f(1-z) + (\beta_u)^2 \bar{F}(z), \quad (21)$$

$$D_d^{\pi^+}(z) = (\beta_u)^2 \bar{F}(z), \quad (22)$$

$$D_s^{\pi^+}(z) = (\beta_{su})^2 \bar{F}(z) = (\beta_s)^2 \bar{F}(z), \quad (23)$$

where we adopt the notation $\beta_s = \beta_{su}$.

In the original Field–Feynman model, the parameters β_u and β_s are identical: $\beta_u = \beta_s$. This means that the effect of producing quark–antiquark pairs excited by the incident s or u quark is assumed to be the same. This leads to the result of $D_d^{\pi^+}(z) = D_s^{\pi^+}(z)$. Indeed, the effect due to the quark mass in the hadronization process strictly speaking cannot be ignored. In other words, the excitation of a quark–antiquark pair by incident quarks with different masses will be different. This can be taken into account by introducing $\beta_u \neq \beta_s$, and thus we obtain $D_u^{\pi^+}(z) \neq D_d^{\pi^+}(z)$. That is, we take into account the difference in the excitation of quark–antiquark pairs by the initial light-flavor (u or d) and strange (s) quarks. Here, $SU(2)$ symmetry is still assumed, $\beta_{qu} = \beta_{qd} = \beta_q$.

3 The fragmentation functions of the pion

From (21) and (22), two relations can be obtained:

$$D_d^{\pi^+}(z)/D_u^{\pi^+}(z) = \frac{\beta_u(1/z - 1)}{(1 - \beta_u + \beta_u/z)}, \quad (24)$$

$$D_u^{\pi^+}(z) - D_d^{\pi^+}(z) = \beta_u f(1-z). \quad (25)$$

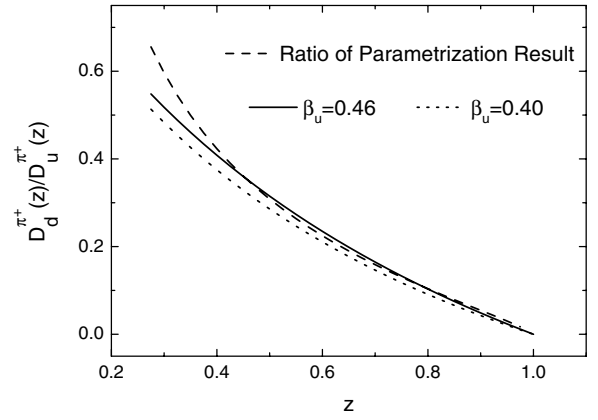


Fig. 2. The dash curve is the ratio of $D_d^{\pi^+}/D_u^{\pi^+}$ from the KLC parameterization [3], and the solid and dot curves are the analytical results of this work with two different values of β_u

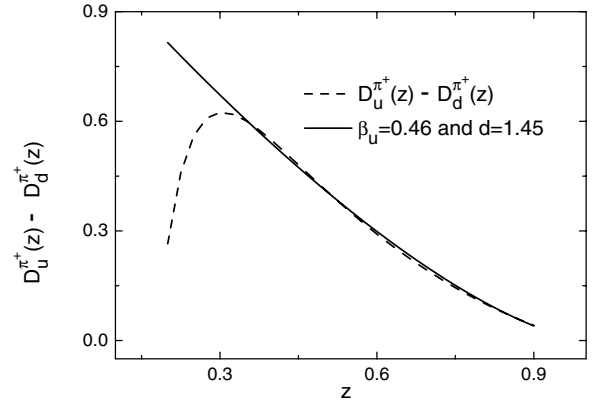


Fig. 3. The dash curve is $D_u^{\pi^+} - D_d^{\pi^+}$ from the KLC parameterization [3], and the solid curve is $\beta_u(d+1)(1-z)^d$ (see (25)) with $\beta_u = 0.46$ and $d = 1.45$

The first relation is independent of $f(z)$, so that it is useful to set the parameter β_u . The parameterization results [3] of $D_u^{\pi^+}(z)$ and $D_d^{\pi^+}(z)$ are used to adjust the parameters. We try to set the value of β_u by plotting $D_d^{\pi^+}(z)/D_u^{\pi^+}(z)$ for the analytical result in this work and the parameterization result by KLC, as shown in Fig. 2. We find that $\beta_u = 0.46$ in the domain of $z \in (0.3, 1)$. We also adopt the option of $\beta_u = 0.4$, which was used in the original model [6], as a comparison. Then plotting the function $D_u^{\pi^+}(z) - D_d^{\pi^+}(z)$ in this work and in the KLC parameterization, we get $d = 1.45$ with $\beta_u = 0.46$, as shown in Fig. 3. It is surprising that we can get compatible curves by only adjusting the parameters β_u and d at $z > 0.3$, as $f(z) = (d+1)z^d$ is only an assumption in the original model. There is no general reason for this choice except for the feasibility of solving the integral equation (5) in an exact way. And now, at some degree, the KLC parameterization results extracted from experimental results without any assumption about favored and unfavored transitions, support the Field–Feynman assumption of $f(z)$ as quite right in the domain of $z \in (0.3, 1)$. Substituting $\beta_u = 0.46$ and $d = 1.45$ into (21) and (22),

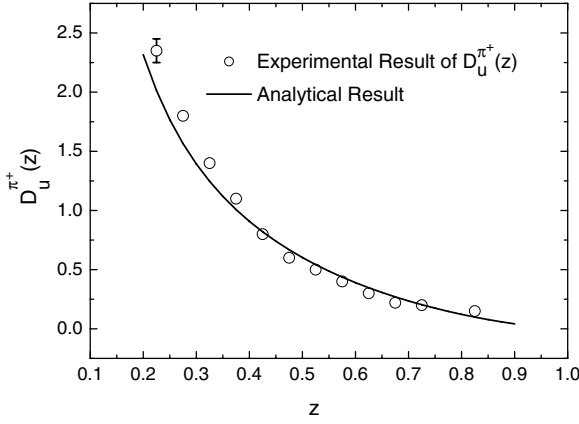


Fig. 4. The circles are the experimental results from [3], and the solid curve is the analytical result of this work

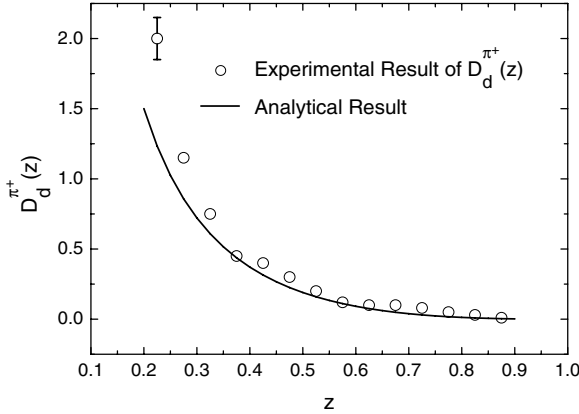


Fig. 5. The circles are the experimental results from [3], and the solid curve is the analytical result of this work

we get the analytical results of $D_u^{\pi^+}(z)$ and $D_d^{\pi^+}(z)$,

$$D_u^{\pi^+}(z) = 1.127(0.54 + 0.46/z)(1-z)^{1.45}, \quad (26)$$

$$D_d^{\pi^+}(z) = 0.51842z^{-1}(1-z)^{2.45}. \quad (27)$$

We compare the analytical results of (26) and (27) with the experimental results in Figs. 4 and 5. It is obvious that the analytical results of (26) and (27) can well explain the experimental results, especially in the domain of $z \in (0.3, 1)$.

We now look at the unfavored fragmentation function $D_s^{\pi^+}(z)$, which was predicted to be the same as the unfavored fragmentation function $D_d^{\pi^+}(z)$ in the original model. We present the experimental results of $D_s^{\pi^+}(z)$ and the analytic result (23) of this work in Fig. 6, with the value of β_s adjusted to 0.38. That means that 38% quark–antiquark pairs in the color field excited by an s quark are the light-flavor $u\bar{u}$ or $d\bar{d}$ pairs, whereas there are 46% $u\bar{u}$ or $d\bar{d}$ quark–antiquark pairs excited by a light-flavor u or d quark. We should notice that $\beta_{ss} \neq \beta_{us}$, as $\beta_{ss} = 1 - 2\beta_{su} = 1 - 2\beta_s = 0.24$ is the fraction of $s\bar{s}$ pairs excited by an s quark, whereas $\beta_{us} = 1 - 2\beta_{uu} = 1 - 2\beta_u = 0.08$ is the fraction of $s\bar{s}$ pairs excited by a u or d quark. This is quite reasonable, considering that there should be a larger

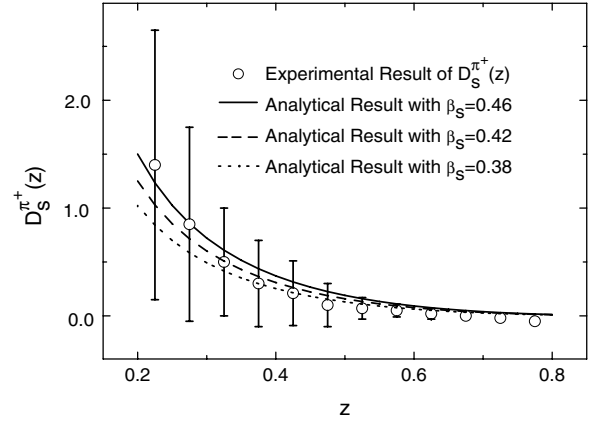


Fig. 6. The circles are the experimental results from [3], and the three curves show three analytical results of this work with different β_s . It goes back to the original Field–Feynman model when $\beta_s = \beta_u = 0.46$

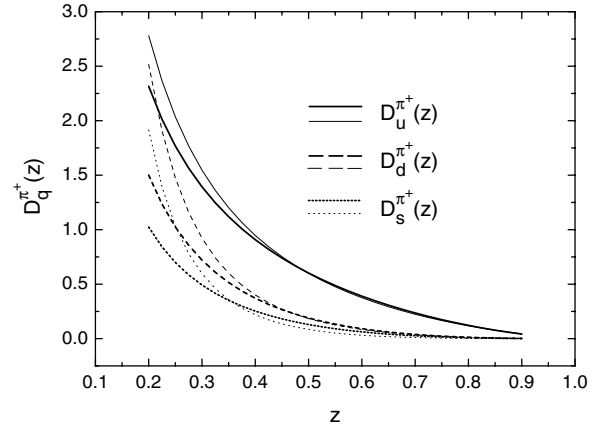


Fig. 7. The fragmentation functions of the pion with distinction between $D_u^{\pi^+}$, $D_d^{\pi^+}$, and $D_s^{\pi^+}$; the thick curves are the analytical results of this work, and the thin curves are the KLC parameterization results [3]

fraction of $s\bar{s}$ pairs excited by an s quark than by a light-flavor (u or d) quark. Therefore we have

$$D_s^{\pi^+}(z) = 0.35378z^{-1}(1-z)^{2.45}. \quad (28)$$

If we choose $\beta_s = 0.46$, $D_s^{\pi^+}(z)$ is identical to $D_d^{\pi^+}(z)$, and we return back to the original model.

We present in Fig. 7 our analytical results for a complete set of fragmentation functions of the pion with distinction between $D_u^{\pi^+}$, $D_d^{\pi^+}$, and $D_s^{\pi^+}$, and compare them with the KLC parameterization results. We find that the two sets of fragmentation functions are compatible, especially at $z > 0.3$. Considering that a large uncertainty may exist in the parameterization of the unfavored fragmentation function $D_d^{\pi^+}(z)$ at small z , we may consider the analytical results in this work as an alternative set of fragmentation functions for possible applications, for example, to check the sensitivity on different fragmentation functions in the predictions of various pion fragmentation processes [5].

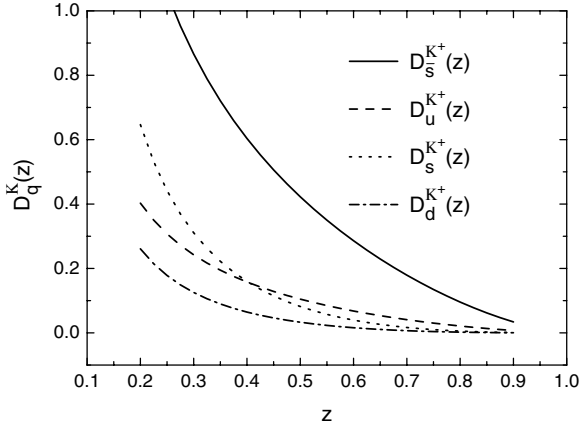


Fig. 8. The fragmentation functions of the kaon in this work with distinction between $D_s^{K^+}(z)$, $D_u^{K^+}(z)$, $D_s^{K^+}(z)$, and $D_d^{K^+}(z)$

One notices from Figs. 2 and 3 that the analytical results differ by those obtained by KLC in the small z region, as also happens for the analytical results and the experimental data in Figs. 4 and 5. This feature is easy to understand: the sea content of the meson has not been taken into account in the Field-Feynman model. The small z behaviors of the fragmentation functions are also influenced by the sea quark-antiquark pairs inside the meson [9]. Thus it is necessary to consider this aspect if one wishes to describe the small z behaviors of fragmentation functions with a more reasonable picture.

4 The fragmentation functions of the kaon

We now apply the same framework to the fragmentation functions of the kaon. Following the same procedures, we get

$$D_s^{K^+}(z) = \beta_s f(1-z) + \beta_s(1-2\beta_s)\bar{F}(z), \quad (29)$$

$$D_u^{K^+}(z) = (1-2\beta_u)f(1-z) + \beta_u(1-2\beta_u)\bar{F}(z), \quad (30)$$

$$D_s^{K^+}(z) = \beta_s(1-2\beta_s)\bar{F}(z), \quad (31)$$

$$D_d^{K^+}(z) = D_u^{K^+}(z) = D_d^{K^+}(z) = \beta_u(1-2\beta_u)\bar{F}(z), \quad (32)$$

where $\beta_s = \beta_{su}$, $\beta_u = \beta_{uu}$, and $\beta_{qs} = 1-2\beta_q$.

With the parameters $\beta_u = 0.46$, $\beta_s = 0.38$, and $d = 1.45$ being the same as for the pion, we get a complete set of fragmentation functions of the kaon:

$$D_s^{K^+}(z) = 0.931(0.76 + 0.24/z)(1-z)^{1.45}, \quad (33)$$

$$D_u^{K^+}(z) = 0.196(0.54 + 0.46/z)(1-z)^{1.45}, \quad (34)$$

$$D_s^{K^+}(z) = 0.22344z^{-1}(1-z)^{2.45}, \quad (35)$$

$$D_d^{K^+}(z) = 0.09016z^{-1}(1-z)^{2.45}. \quad (36)$$

These analytical results of the fragmentation functions of the kaon are plotted in Fig. 8 with four independent curves: $D_s^{K^+}(z)$, $D_u^{K^+}(z)$, $D_s^{K^+}(z)$, and $D_d^{K^+}(z)$. There

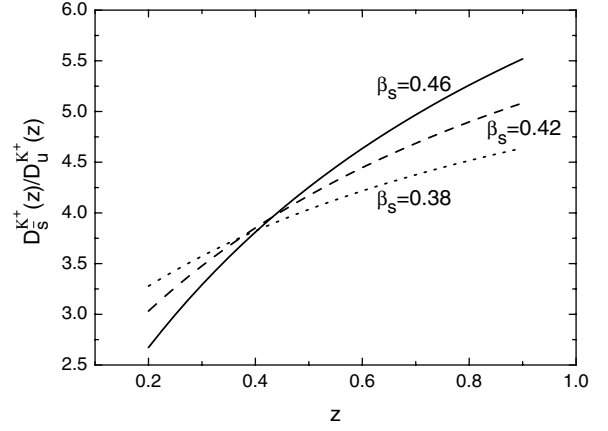


Fig. 9. These three curves give the ratio between the favored fragmentation functions $D_s^{K^+}(z)/D_u^{K^+}(z)$ in this work. It goes back to the original Field-Feynman model when $\beta_s = \beta_u = 0.46$

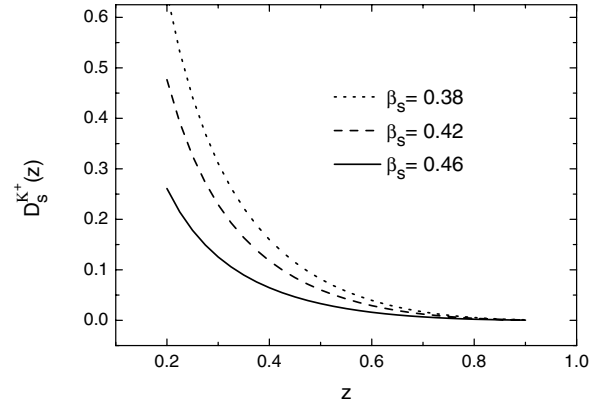


Fig. 10. These three curves show the unfavored fragmentation function $D_s^{K^+}(z)$ with different β_s in this work. It goes back to the original Field-Feynman model when $\beta_s = \beta_u = 0.46$

should be only three independent curves without the difference between β_u and β_s , e.g., $D_s^{K^+}(z)$ equals $D_d^{K^+}(z)$ as in the original model. In principle, the flavor structure of the kaon fragmentation functions is more complicated than that of the pion, as revealed by the Field-Feynman model. However, some available parameterizations [10–13] usually make only a distinction between the favored fragmentation functions, which are related to the valence quarks of the kaon, and the unfavored fragmentation functions, which are related to the light-flavor sea quarks of the kaon.

The effect of introducing $\beta_s \neq \beta_u$ might be small in the fragmentation functions of the pion, but it is amplified in the fragmentation functions of the kaon. It does not only introduce the difference between the unfavored fragmentation functions $D_s^{K^+}(z)$ and $D_d^{K^+}(z)$, but also causes a modification to the favored fragmentation functions $D_s^{K^+}(z)$. In Fig. 9, we plot the ratio of the favored fragmentation functions: $D_s^{K^+}(z)/D_u^{K^+}(z)$. The curve with $\beta_s = 0.42$ is adopted as an alternative option for comparison, and the effect of β_s on the unfavored frag-

mentation function is illustrated in Fig. 10. The curve with $\beta_s = 0.46$ is the result in the original model, where $\beta_s = \beta_u = 0.46$. Therefore we notice a significant modification to the favored fragmentation function $D_s^{K^+}(z)$ in comparison with that in the original model. We also plot the prediction of the unfavored fragmentation function $D_s^{K^+}(z)$, and find that it differs from $D_d^{K^+}(z)$ significantly. These predictions can be tested by future experiments, such as the semi-inclusive kaon productions in deep inelastic scattering by the HERMES collaboration and the CLAS collaboration et al. The parameters used in this work can also be constrained by further detailed studies, or the framework should be extended to a more general situation with more sophisticated aspects taken into account.

5 Summary

In summary, we analyzed the fragmentation functions of both the pion and the kaon in the Field–Feynman recursive model with a significant modification. By taking into account the difference between the excitation of sea quark–antiquark pairs by the initial light-flavor (u or d) and strange (s) quarks, we can distinguish between $D_u^{\pi^+}$, $D_d^{\pi^+}$, and $D_s^{\pi^+}$ in the Field–Feynman model with only three parameters. The analytical results obtained are compatible with the available empirical results, and may serve as an alternative set of completed fragmentation functions of the pion. We also extended the same framework to the kaon, and provided a new set of fragmentation functions of the kaon with a clear distinction between $D_s^{K^+}(z)$, $D_u^{K^+}(z)$, $D_s^{K^+}(z)$, and $D_d^{K^+}(z)$. Thus, this work provides the fragmentation functions of the pion and the kaon with a same set of only three parameters. The predictions can be tested by future experiments on the fragmentation functions of the pion and the kaon.

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